

GPS Relativity

Predicting and Measuring the Satellite-Clock Correction

$$dt_r = F e \sqrt{A} \sin E_k$$

$$F = -2\sqrt{\mu}/c^2 = -4.442807633 \times 10^{-10} \text{ s}/\sqrt{m}$$

Experiment 1 (prediction) — relativistic clock wobble from a broadcast ephemeris only. Amplitudes match $|F|e\sqrt{A}$ to $<0.1\%$.

Experiment 2 (measurement) — RTKLIB processed twice (stock vs relativity-removed); residual difference B–A tracks the predicted $c \cdot dt_r$ with correlation 0.98–0.99 for high-eccentricity satellites.

Station POTS (Potsdam) • GPS • 2026 DOY 199–201

Both experiments succeeded.

Full console output (analyze.py --exp both)

=== EXPERIMENT 1: predicted relativistic clock wobble ===

Loaded ephemerides for 32 GPS satellites from ['BRDC_199.rnx', 'BRDC_200.rnx', 'BRDC_201.rnx']

PRN	ecc	sqrtA(m ⁵)	pred_amp(ns)	pred_amp(m)	meas_amp(ns)
G28	0.0005196	5153.626	1.190	0.3567	1.190
G21	0.0007922	5153.590	1.814	0.5438	1.815
G01	0.0018238	5153.605	4.176	1.2519	4.174
G20	0.0019062	5153.612	4.365	1.3085	4.366
G13	0.0022749	5153.591	5.209	1.5615	5.210
G11	0.0024805	5153.649	5.680	1.7027	5.672
G29	0.0036612	5153.687	8.383	2.5132	8.397
G09	0.0036630	5153.697	8.387	2.5144	8.382
G06	0.0036925	5153.724	8.455	2.5346	8.459
G04	0.0038122	5153.673	8.729	2.6168	8.718
G05	0.0056036	5153.775	12.831	3.8465	12.835
G18	0.0062880	5153.625	14.397	4.3162	14.391
G23	0.0064350	5153.729	14.734	4.4172	14.726
G03	0.0069717	5153.768	15.963	4.7857	15.966
G14	0.0073540	5153.657	16.838	5.0480	16.835
G30	0.0082208	5153.686	18.823	5.6430	18.825
G12	0.0087115	5153.704	19.947	5.9798	19.947
G32	0.0094534	5153.571	21.645	6.4889	21.641
G31	0.0109082	5153.636	24.976	7.4876	24.971
G10	0.0111262	5153.649	25.475	7.6373	25.471
G26	0.0111616	5153.580	25.556	7.6615	25.553
G08	0.0116776	5153.744	26.738	8.0159	26.733
G19	0.0117642	5153.754	26.937	8.0754	26.947
G22	0.0118492	5153.722	27.131	8.1337	27.135
G17	0.0126110	5153.671	28.875	8.6565	28.866
G25	0.0129009	5153.667	29.539	8.8555	29.538
G27	0.0143264	5153.670	32.803	9.8340	32.800
G16	0.0150341	5153.713	34.424	10.3199	34.422
G02	0.0168919	5153.734	38.677	11.5952	38.681
G15	0.0170318	5153.756	38.998	11.6913	39.000
G24	0.0184121	5153.638	42.158	12.6385	42.157
G07	0.0209372	5153.588	47.939	14.3717	47.939

Saved: exp1_wobble_all_sats.png, exp1_wobble_highlight.png, exp1_amplitude_vs_ecc.png

=== EXPERIMENT 2: measured B-A residual vs predicted c*dt_r ===

stat A: ['A_199.pos.stat', 'A_200.pos.stat', 'A_201.pos.stat']

stat B: ['B_199.pos.stat', 'B_200.pos.stat', 'B_201.pos.stat']

Saved: exp2_residual_diff_vs_predicted<sat>.png for ['G07', 'G02', 'G28']

Saved: exp2_position_error_comparison.png

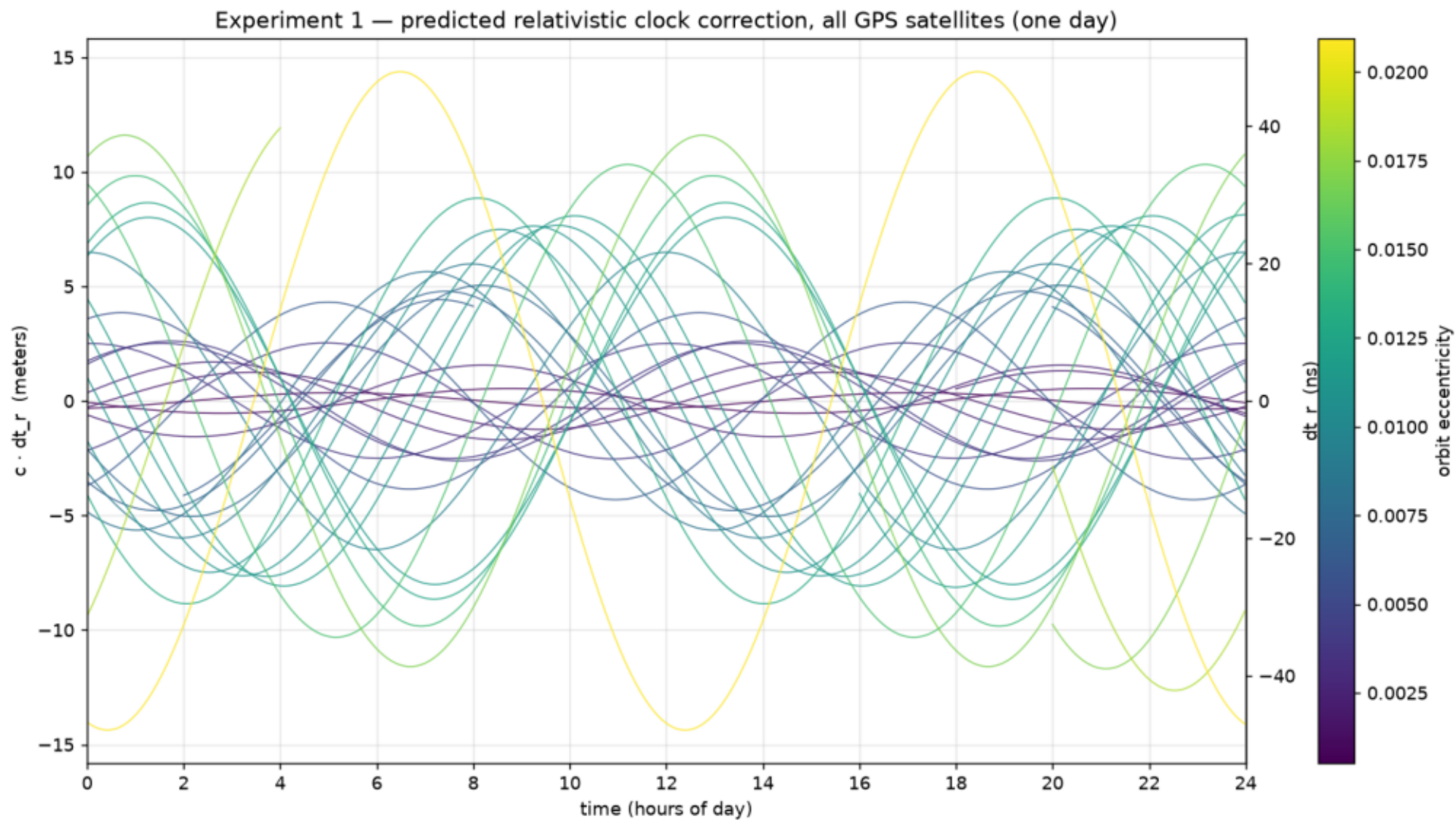
Match statistics (sats with > 2 h of data):

sat	ecc	n_pts	hours	corr	RMS(pred)	RMS(Δ -pred)	ratio	
G01	0.00182	2145	53.8	0.97	2.815	0.643	0.23	
G02	0.01689	2217	54.0	0.99	3.261	0.375	0.12	<-- highlight
G03	0.00697	2184	53.9	0.99	2.768	0.344	0.12	
G04	0.00381	2068	53.6	1.00	2.015	0.229	0.11	
G05	0.00560	2682	65.8	0.92	2.535	0.988	0.39	
G06	0.00369	2811	72.0	0.94	2.953	0.912	0.31	
G07	0.02094	2562	61.7	0.98	4.766	1.130	0.24	<-- highlight
G08	0.01168	2406	61.5	0.97	4.702	1.168	0.25	
G09	0.00366	2165	53.9	1.00	1.834	0.150	0.08	
G10	0.01113	2736	65.5	0.88	2.927	1.159	0.40	
G11	0.00248	2853	72.0	0.98	2.810	0.336	0.12	
G12	0.00871	2193	72.0	0.72	2.760	2.307	0.84	
G14	0.00735	2599	61.6	0.56	2.665	2.379	0.89	
G15	0.01703	404	25.6	0.99	7.757	0.501	0.06	

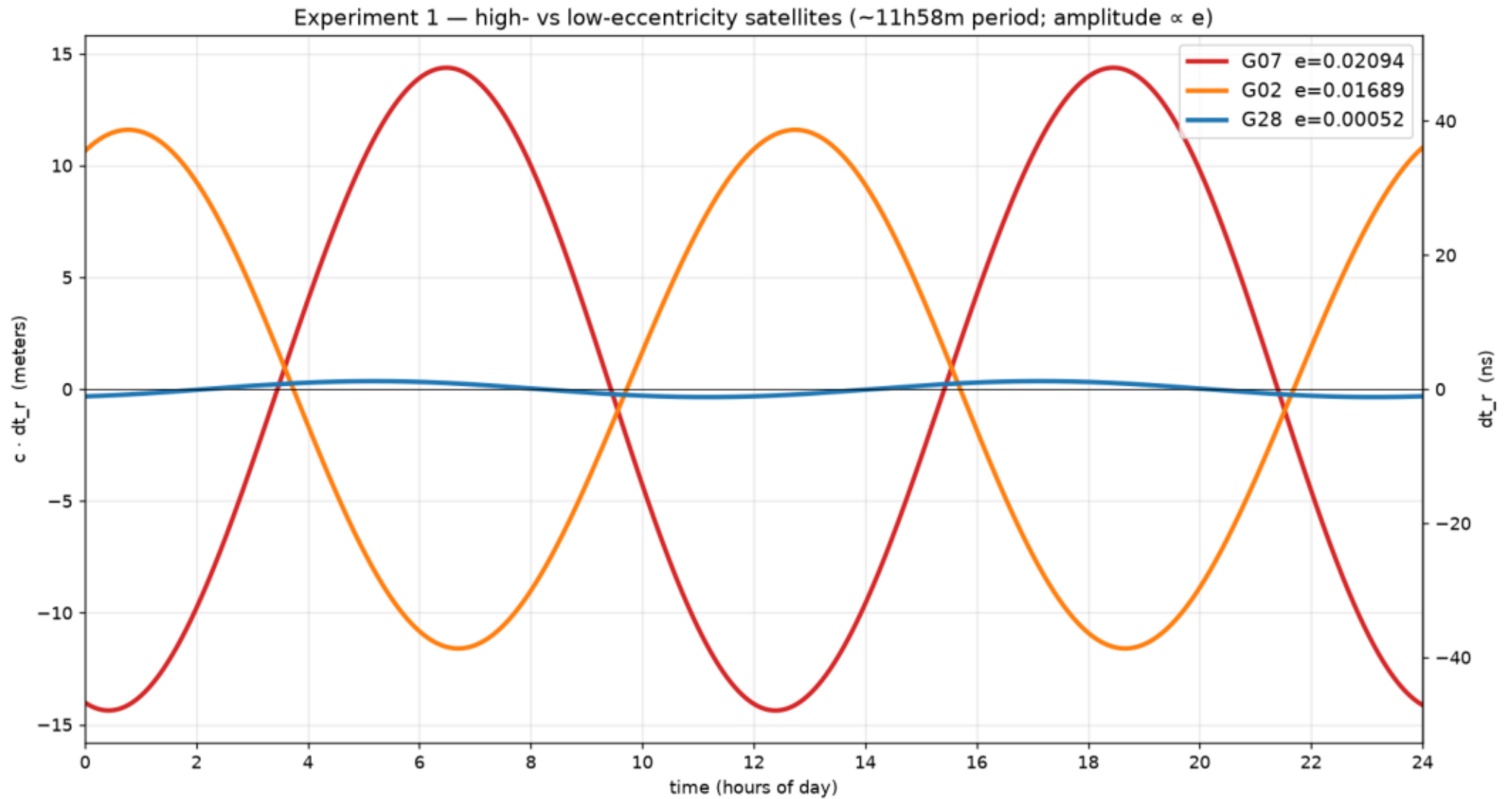
G16	0.01503	2592	61.5	0.96	3.061	0.892	0.29	
G17	0.01261	2801	61.6	0.88	4.807	1.816	0.38	
G18	0.00629	2500	64.9	0.84	3.168	1.490	0.47	
G19	0.01176	2806	61.5	0.80	5.017	1.810	0.36	
G20	0.00191	1226	51.0	0.86	2.746	1.760	0.64	
G21	0.00079	2595	65.2	0.88	1.894	0.885	0.47	
G22	0.01185	1409	53.0	0.83	3.199	1.570	0.49	
G23	0.00644	1374	52.5	0.95	2.208	0.794	0.36	
G24	0.01841	1080	51.6	0.98	5.987	0.667	0.11	
G25	0.01290	2186	72.0	0.97	4.634	1.297	0.28	
G26	0.01116	2483	61.5	0.99	3.036	0.434	0.14	
G27	0.01433	2061	53.6	0.98	3.764	1.240	0.33	
G28	0.00052	2782	72.0	0.99	3.223	0.389	0.12	<-- highlight
G29	0.00366	2160	72.0	0.97	3.053	0.945	0.31	
G30	0.00822	2043	53.8	0.73	2.079	1.675	0.81	
G31	0.01091	2796	72.0	0.99	2.736	0.327	0.12	
G32	0.00945	2881	72.0	0.92	2.635	1.108	0.42	

High-eccentricity sats (e>0.008): median correlation = 0.967, 11/17 exceed 0.9

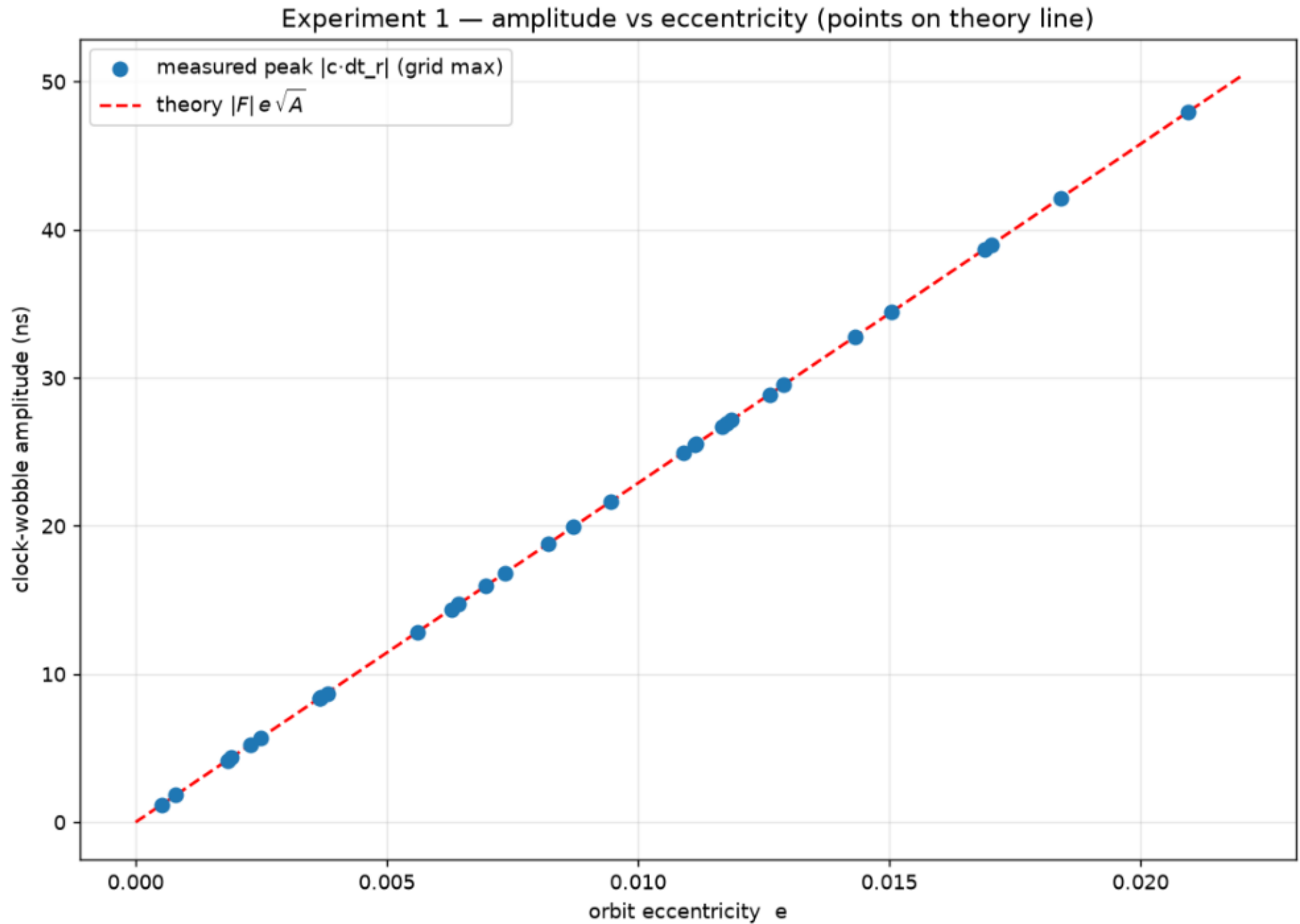
Exp 1 — $c \cdot dt_r$ for all 32 GPS satellites (colored by eccentricity)



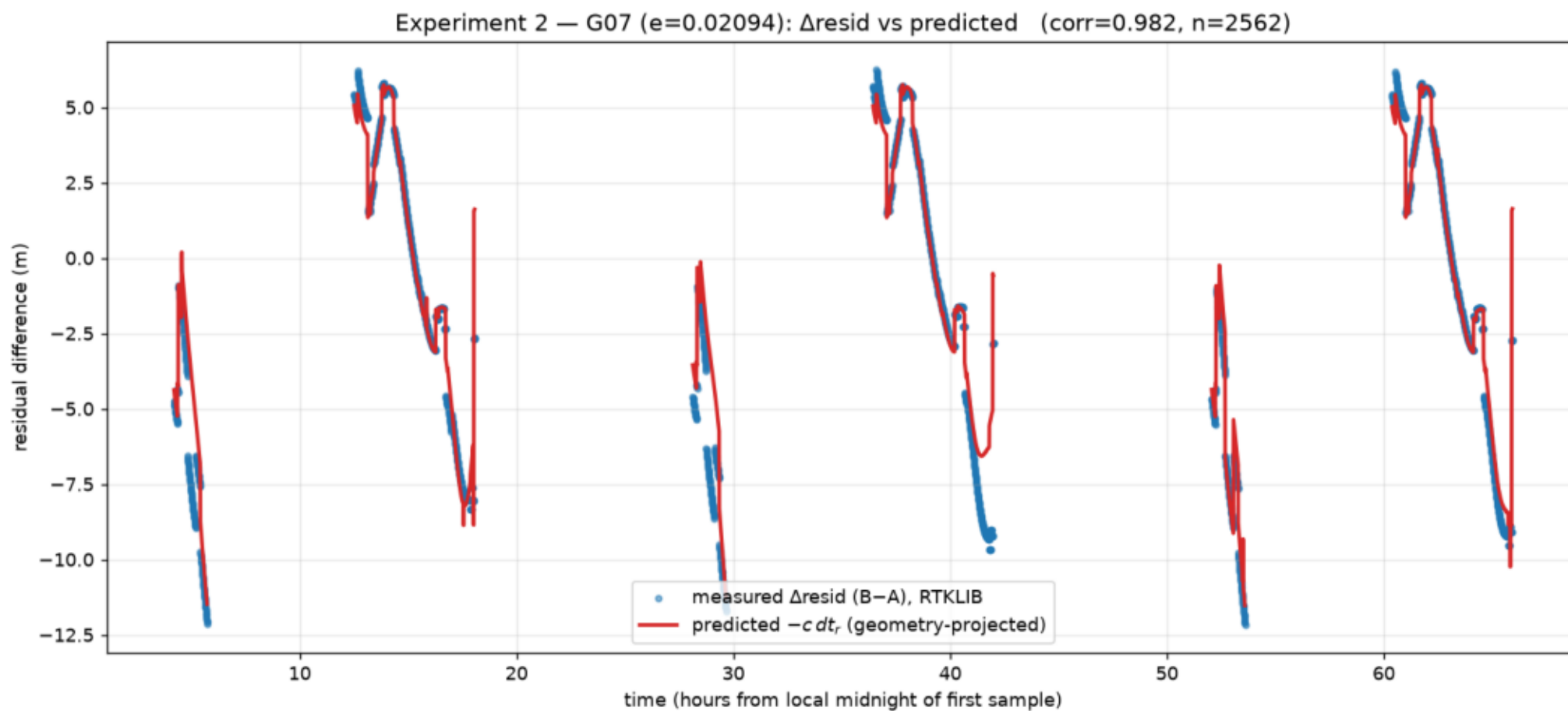
Exp 1 — high-e (G07, G02) vs low-e (G28): amplitude $\propto$ eccentricity



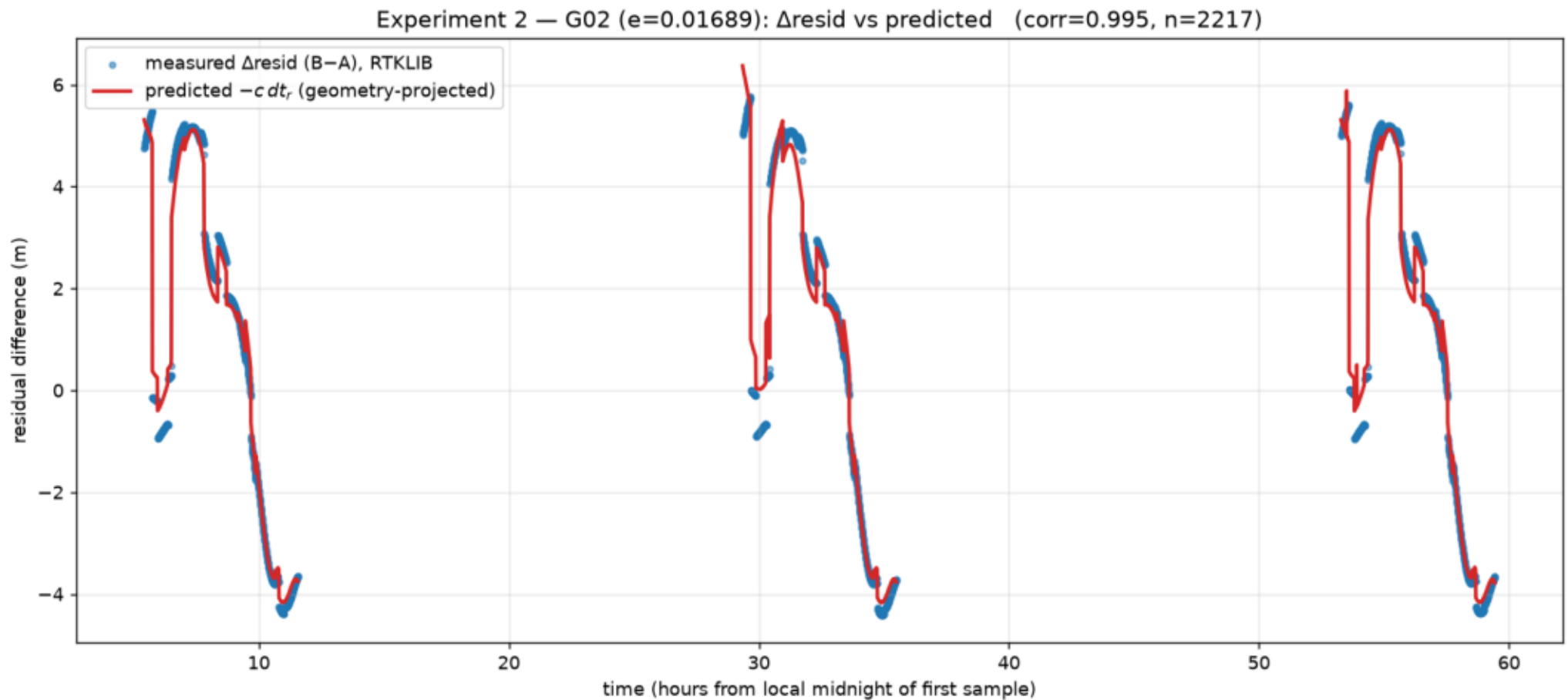
Exp 1 — measured amplitude vs eccentricity, on the $|F|e\sqrt{A}$ theory line



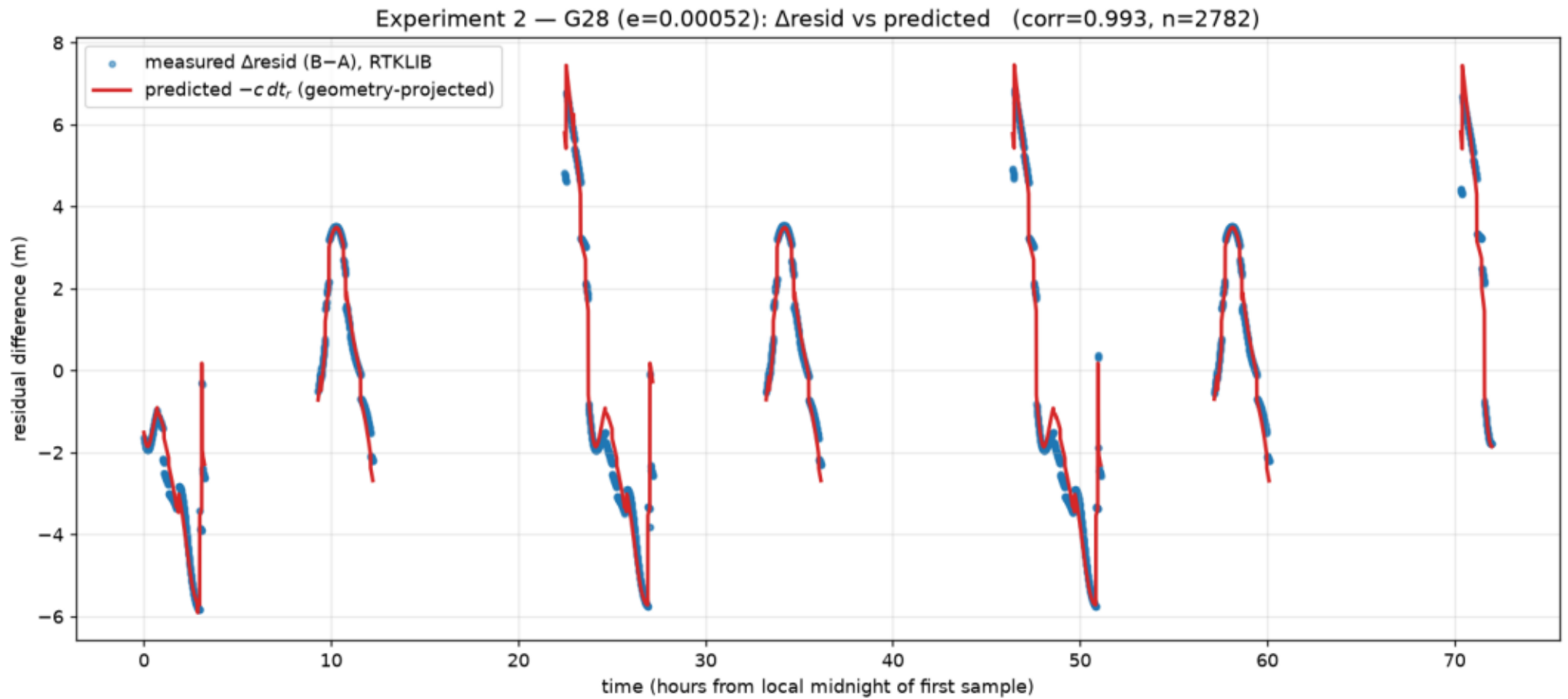
Exp 2 — G07 (e=0.021): measured Δresid vs predicted (corr 0.98)



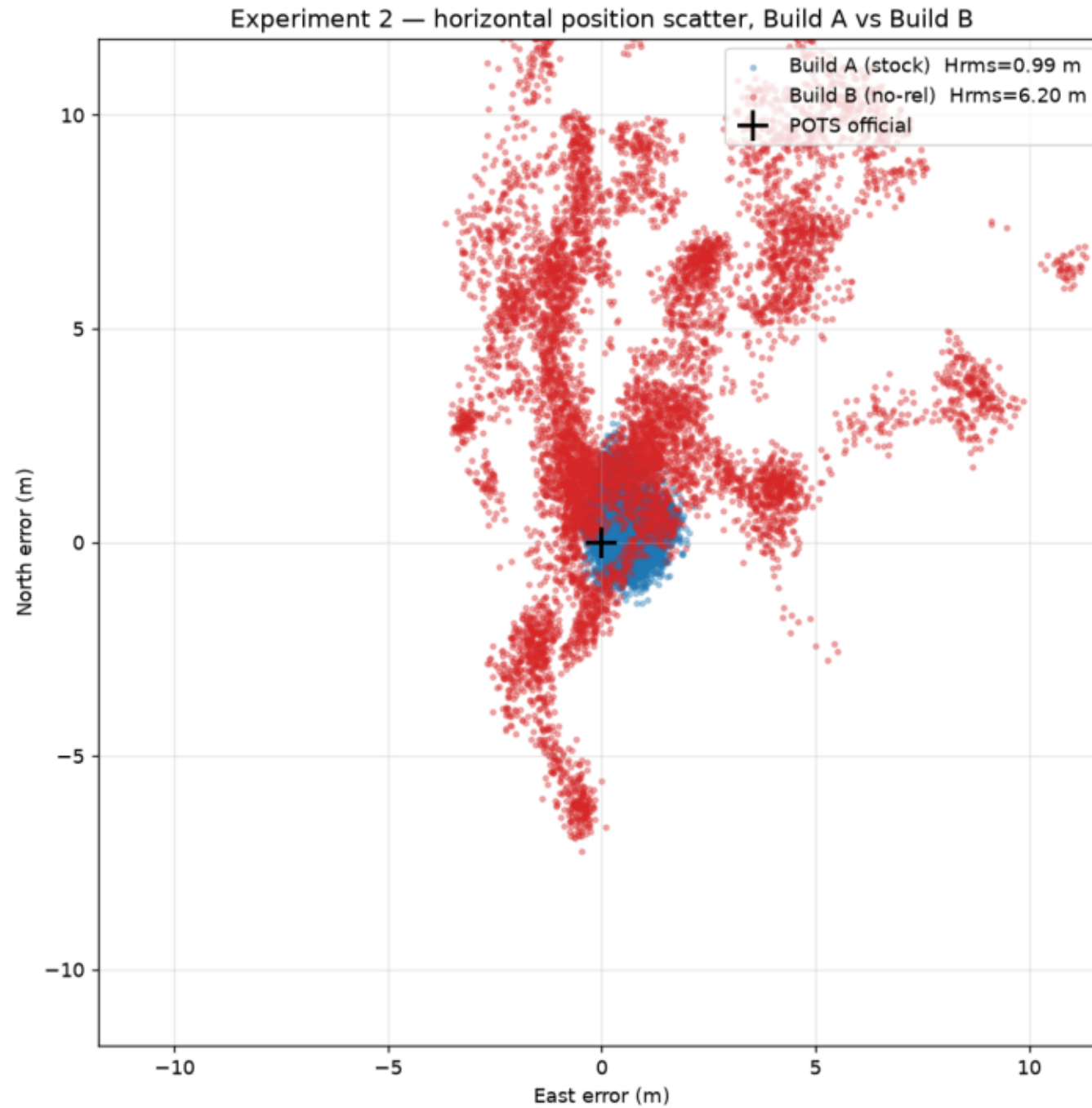
Exp 2 — G02 (e=0.017): measured Δresid vs predicted (corr 0.99)



Exp 2 — G28 (e=0.0005): low-eccentricity control



Exp 2 — horizontal position: Build A 0.99 m vs Build B 6.20 m RMS



patch_norel.diff (relativity correction removed, 3 sites)

```
diff --git a/src/ephemeris.c b/src/ephemeris.c
index 391f45c..d233a0a 100644
--- a/src/ephemeris.c
+++ b/src/ephemeris.c
@@ -284,7 +284,7 @@ extern void eph2pos(gtime_t time, const eph_t *eph, double *rs, double *dts,
    *dts=eph->f0+eph->f1*tk+eph->f2*tk*tk;

    /* relativity correction */
-   *dts-=2.0*sqrt(mu*eph->A)*eph->e*sinE/SQR(CLIGHT);
+   *dts-=0.0*2.0*sqrt(mu*eph->A)*eph->e*sinE/SQR(CLIGHT); /* REL-PATCH: relativistic sat-clock correction removed */

    /* position and clock error variance */
    *var=var_uraeph(sys,eph->sva);
@@ -689,7 +689,7 @@ static int satpos_ssr(gtime_t time, gtime_t teph, int sat, const nav_t *nav,
    dts[1]=eph->f1+2.0*eph->f2*tk;

    /* relativity correction */
-   dts[0]-=2.0*dot3(rs,rs+3)/CLIGHT/CLIGHT;
+   dts[0]-=0.0*2.0*dot3(rs,rs+3)/CLIGHT/CLIGHT; /* REL-PATCH: relativistic sat-clock correction removed */
}
/* radial-along-cross directions in ecef */
if (!normv3(rs+3,ea)) return 0;
diff --git a/src/preceph.c b/src/preceph.c
index 0777c9d..dfe40d7 100644
--- a/src/preceph.c
+++ b/src/preceph.c
@@ -842,7 +842,7 @@ extern int pep2pos(gtime_t time, int sat, const nav_t *nav, int opt,
}
/* relativistic effect correction */
if (dtss[0]!=0.0) {
-   dts[0]=dtss[0]-2.0*dot3(rs,rs+3)/CLIGHT/CLIGHT;
+   dts[0]=dtss[0]-0.0*2.0*dot3(rs,rs+3)/CLIGHT/CLIGHT; /* REL-PATCH: relativistic sat-clock correction removed */
    dts[1]=(dtst[0]-dtss[0])/tt;
}
else { /* no precise clock */
```